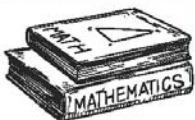


Name :

Class :



# Trigonometric Identities



A Trigonometric Identity is an equation involving the trigonometric functions that holds for all values of the variable.

## Trigonometric Identities

### Reciprocal Identities

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} \\ \sec \theta &= \frac{1}{\cos \theta} \\ \cot \theta &= \frac{1}{\tan \theta} \end{aligned}$$

### Pythagorean Identities

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \sec^2 \theta &= 1 + \tan^2 \theta \\ \csc^2 \theta &= 1 + \cot^2 \theta \end{aligned}$$

### Half-Angle Formulas

$$\begin{aligned} \sin\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \cos\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan\left(\frac{\theta}{2}\right) &= \frac{1 - \cos \theta}{\sin \theta} \end{aligned}$$



Prove the following Trigonometric Identities

$$\begin{aligned} 1) \quad \frac{\tan^2 \theta - \sec^2 \theta}{\cos \theta} &= -\sec \theta \\ \text{L.H.S} & \\ &= \frac{\tan^2 \theta - (1 + \tan^2 \theta)}{\cos \theta} \quad 1 + \tan^2 \theta = \sec^2 \theta \\ &= \frac{-1}{\cos \theta} \\ &= -\sec \theta \\ &= \text{R.H.S} \end{aligned}$$

$$\begin{aligned} 2) \quad 1 - \sin^2 \theta &= \frac{\sin^2 \theta}{\tan^2 \theta} \\ \text{R.H.S} &= \frac{\sin^2 \theta}{\tan^2 \theta} \\ &= \frac{\sin^2 \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \cos^2 \theta \quad \sin^2 \theta + \cos^2 \theta = 1 \\ &= 1 - \sin^2 \theta = \text{L.H.S} \end{aligned}$$

$$\begin{aligned} 3) \quad \tan \theta \cos \theta &= \sin \theta \\ \text{L.H.S} & \\ &= \frac{\sin \theta}{\cos \theta} \cos \theta \\ &= \sin \theta \\ &= \text{R.H.S} \end{aligned}$$

$$\begin{aligned} 4) \quad \frac{\sin^2 \theta}{1 - \cos \theta} &= 1 + \cos \theta \\ \text{L.H.S} & \quad \sin^2 \theta + \cos^2 \theta = 1 \\ &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\ &= \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta} \\ &= 1 + \cos \theta \\ &= \text{R.H.S} \end{aligned}$$

$$\begin{aligned} 5) \quad \tan \theta \csc \theta &= \sec \theta \\ \text{L.H.S} & \\ &= \frac{\sin \theta}{\cos \theta} \frac{1}{\sin \theta} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \\ &= \text{R.H.S} \end{aligned}$$

$$\begin{aligned} 6) \quad \cos^2 \theta &= \sin^2 \theta + 2 \cos^2 \theta - 1 \\ \text{R.H.S} & \quad \sin^2 \theta + \cos^2 \theta = 1 \\ &= 2 \cos^2 \theta + 1 - \cos^2 \theta - 1 \\ &= \cos^2 \theta \\ &= \text{L.H.S} \end{aligned}$$